JOURNAL OF GUIDANCE, CONTROL, AND DYNAMICS
Vol. 32, No. 1, January–February 2009

Engineering Notes

ENGINEERING NOTES are short manuscripts describing new developments or important results of a preliminary nature. These Notes should not exceed 2500 words (where a figure or table counts as 200 words). Following informal review by the Editors, they may be published within a few months of the date of receipt. Style requirements are the same as for regular contributions (see inside back cover).

Robust Control Design for Linear Systems Using an Ecological Sign-Stability Approach

Rama K. Yedavalli*
The Ohio State University, Columbus, Ohio 43210

DOI: 10.2514/1.35763

I. Introduction

THE fields of population biology and ecology deal with the analysis of growth and decline of populations in nature and the struggle of species to predominate over one another. Many mathematical population models were proposed over the last few decades, with the most significant contributions coming from the work of Lotka [1] and Volterra [2]. The predator–prey models of Lotka and Volterra, studied extensively by ecologists and population biologists, consists of a set of nonlinear ordinary differential equations, and stability of the equilibrium solutions of these models has been a subject of intense study for students of life sciences [3–6]. For example, many standard textbooks on mathematical models in biology, such as [7], cover these issues. These small perturbations from equilibrium can be modeled as linear state-space systems in which the state-space plant matrix is the Jacobian, and it is important to analyze the stability of these state-space (Jacobian) matrices. For communities of five or more species, the order of these matrices is high enough to cause difficulties in assessing the stability. For this reason, to circumvent these difficulties, alternative concepts of reduced computation have been proposed, and one such important concept is that of qualitative (or sign) stability. The technique of qualitative stability applies ideally to large-scale systems in which there is no quantitative information about the interrelationship of species or subsystems. The motivation for this method actually came from economics. The paper by economists Quirk and Ruppert [8] was later followed by further research and application to ecology by May [9] and Jeffries [10]. Note that in a complex community composed of many species, numerous interactions take place. The magnitudes of the mutual effects of species on each other are seldom accurately known, but one can establish with greater certainty whether predation, competition, or other influences are present. This means that in the Jacobian matrix, one does not technically know the actual magnitudes of the partial derivatives, but their signs are known with certainty. Thus, the qualitative information about the species is represented by the signs $+$, $-$, or 0. Thus, the (i, j)th entry of the state-space (Jacobian) matrix simply consists of signs $+$, $-$, or 0, with the positive sign indicating that species j has a positive influence on species i, the negative sign indicating a negative influence, and zero indicating no influence. An alternative visual representation of this situation can also be given by a directed graph (or, simply, a digraph), as shown in Fig. 1. For example, with respect to the digraph of Fig. 1a, the corresponding sign-pattern matrix is given by

$$A = \begin{bmatrix} 0 & + & + \\ - & 0 & 0 \\ - & 0 & - \end{bmatrix}$$

The question then is whether or not it can be concluded, just from this sign pattern, that the system is stable. If so, we say the system is *qualitatively stable* [11–13]. In some literature, this concept is also labeled as *sign stability*. In what follows, we use these two terms interchangeably. It is important to keep in mind that systems (matrices) that are qualitatively (sign) stable are also stable in the ordinary sense. That is, qualitative stability implies Hurwitz stability in the ordinary sense of engineering sciences. *In other words, once a particular sign matrix is shown to be qualitatively (sign) stable, we can insert numerical values of any magnitudes in those entries, and for all those values, the matrix is automatically Hurwitz stable*. This is the most attractive feature of a sign-stable matrix. However, the converse is not true. Systems that are not qualitatively stable can still be stable in the ordinary sense for certain appropriate magnitudes in the entries. From now on, to distinguish from the concept of qualitative stability of life sciences literature, we use the label of quantitative stability for the standard Hurwitz stability in engineering sciences.

It is important to note that the general labels of qualitative stability and sign stability are also used in engineering sciences literature [14]. For example, [14] briefly discusses sign stability in the context of matrix diagonal stability in systems and computation and cites many other works. However, these references touch upon the sufficient conditions for sign stability but do not allude to the *color-test* conditions that are part of the necessary and sufficient conditions provided in the ecology literature. Also, the qualitative-stability concept discussed in the nonlinear systems literature of engineering sciences is not the same as the qualitative stability of ecology. This is because in the former case, the systems considered include time-varying systems that include the concept of quadratic stability, whereas in the latter case, only strict real linear time-invariant systems (in which stability means eigenvalues having strictly negative real parts) are considered. For this reason, the qualitative (sign) stability literature from ecology presents the necessary and sufficient conditions in terms of ecological terms involving the color test and there is no equivalent test reported in the engineering sciences literature.

One of the contributions of this Note and [15] is to transform this color test in matrix notation so that it can be easily used by the engineering sciences community. It is worth reiterating that the major difference between the literature of qualitative (sign) stability of ecology and that of engineering sciences is that in the former case, one can decide a priori, using the necessary and sufficient conditions stated in that literature, which sign-pattern matrices of a given order are sign-stable and which are not. Thus, one can store a priori all the 3 by 3 sign-stable matrices, all the 4 by 4 sign-stable matrices, and so on. According to this author, this type of a priori knowledge of the role played by the different signs of a given-order matrix is not

Presented as Paper 6262 at the AIAA Guidance, Navigation, and Control Conference, San Francisco, CA, 15–18 August 2005; received 19 November 2007; accepted for publication 30 July 2008.

*Professor, Department of Aerospace Engineering, 2036 Neil Avenue; yedavalli.1@osu.edu. Associate Fellow AIAA.

currently available in the current engineering sciences literature. Hence, to emphasize these differences in the use of the phrase *sign stability* in engineering sciences and ecology, we henceforth use the phrase *ecological sign stability* for the sign-stability ideas from ecology literature.

With this motivation and backdrop, the Note is organized as follows. In Sec. II, we review the conditions for ecological sign stability, along with a few examples to illustrate the application of these conditions. Then in Sec. III, we extend these ideas to synthesize controllers via the ecological sign-stability approach and show that the resulting controllers are robust for all sign-preserving parameter variations. We also illustrate the proposed technique by applying it to a three-axis satellite attitude stabilization problem. Finally, Sec. IV offers some concluding remarks, elaborating on the possible avenues for extending these ideas to various other problems in engineering sciences.

II. Review of Conditions for Ecological Sign Stability

We now present the necessary and sufficient conditions for qualitative stability as given by May [9] and Jeffries [10]. Let A be the matrix with entries a_{ij}. The following are necessary conditions for sign stability. $M1$ states that $a_{ii} \leq 0$ for all i. $M2$ states that $a_{ii} < 0$ for at least one i. $M3$ states that $a_{ij}a_{ji} \leq 0$ for all $i \neq j$. $M4$ states that $a_{ij}a_{jk}, \ldots, a_{qr}a_{ri} = 0$ for any sequences of three or more distinct indices $i, j, k, \ldots, q, r$. $M5$ states that $\det A \neq 0$.

In ecological and population dynamics terms, these conditions can be interpreted as follows [7]. For $M1$, no species exerts positive feedback on itself. For $M2$, at least one species is self-regulating. For $M3$, the members of any given pair of interacting species must have opposite effects on each other. For $M4$, there are no closed chains of interactions among three or more species. For $M5$. there is no species that is unaffected by interactions with itself or with other species.

As also discussed in [7], if the information is given in term-directed signal graphs, then the following conditions are equivalent to the preceding conditions. For $M1$, there are no $+$ loops on any single species (that is, no positive feedback). For $M2$, there is at least one $-$ loop on some species in the graph. For $M3$, there are no pair of like arrows connecting a pair of species. For $M4$, there are no cycles connecting three or more species. For $M5$, no node is devoid of input arrows.

Note that the preceding conditions are only *necessary* conditions for qualitative stability. However, Jeffries [10] developed *necessary and sufficient* conditions for qualitative stability by devising an auxiliary set of conditions, which he called the *color test*, that replaces condition $M2$. Before describing the color test, it is important to gather some definitions as follows:

Definition 1: A *predation link* is a pair of species connected by one $+$ line and one $-$ line.

Definition 2: A *predation community* is a subgraph consisting of all interconnected predation links.

If one defines a species not connected to any other predation link as a *trivial* predation community, then it is possible to decompose any graph into a set of distinct predation communities. For example, the systems shown in Fig. 1 have predation communities as follows: (2,1,3) (Fig. 1a); (H_1, P_1), (H_2, P_2), and (H_3, P_3) (Fig. 1b); and (1,2,3,4,5) (Fig. 1c).

The following color scheme constitutes the test to be made. A predation community is said to *fail the color test* if it is not possible to color each node in the subgraph black or white in such a way that each self-regulating node is black, there is at least one white point, each white point is connected by a predation link to at least one other white point, and each black point is connected by a predation link to one white node that is also connected by a predation link to one other white node.

Jeffries [10] proved that for qualitative stability (i.e., asymptotic stability with only signs as elements), a community must satisfy the main conditions $M1$, $M3$, $M4$, and $M5$ and, in addition, must have only predation communities that *fail* the color test.

Because these necessary and sufficient conditions for qualitative stability are given in ecological (population dynamics) terms, it takes

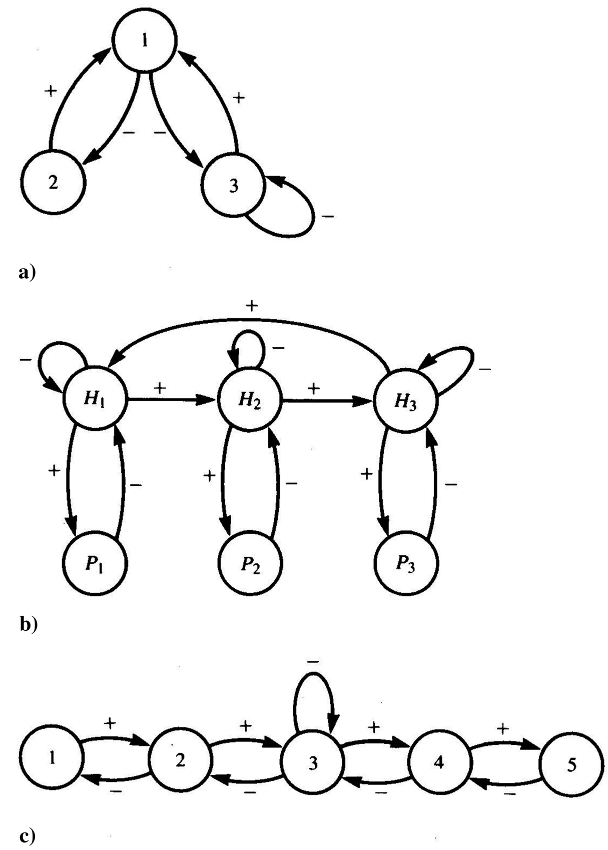

Fig. 1 Signed digraphs equivalent to the matrix representation of sign patterns.

a little effort to state them in matrix theory notation. Out of these, main conditions $M1$, $M2$, $M3$, $M4$, and $M5$ are already amenable to matrix theory notation, but the *color test* was stated in ecological terms. As mentioned earlier, even though [14] discusses qualitative-stability conditions briefly in the context of matrix diagonal stability, the conditions discussed are only sufficient conditions and it does not allude to the role played by the color test in the necessary and sufficient conditions. Note that the transformation of these color-test conditions in matrix theory notation as done in this Note is very beneficial to the engineering community. As far as this author's literature search is concerned, until now, there was no evidence of these qualitative-stability conditions, *along with the color test,* appearing in engineering sciences literature in the standard matrix theory notation. Another main result presented in this Note is the concept of robust stabilization using sign stability.

A. Color-Test Conditions in Terms of Matrix Element Notation

To begin testing the necessary and sufficient conditions (along with the color test), it is recommended that the main necessary conditions $M1$ through $M5$ be tested. Fortunately, these conditions are already in matrix theory notation. It needs to be emphasized that we go to the color test only after satisfying the main conditions $M1$ through $M5$. Note that $M4$ does not apply to 2 by 2 matrices. Note that if all the diagonal elements are negative, there is no need to go to the color test: it automatically fails.

To translate the color-test conditions in matrix theory notation, we need to consider irreducible matrices to reflect the ecological notion of trivial and regular predation communities as defined in [10] through Definition 1 and Definition 2 stated before. Note that Definition 2 implies that a predation community that consists of all interconnected predation links is strongly connected. A strongly connected digraph is the geometric/visual representation of an irreducible matrix or an indecomposable matrix. Because the color test is applicable only for distinct predation communities, the matrices that can be checked for sign stability using the color test must all be irreducible.

For completeness, we now provide the formal definition of an irreducible matrix from the matrix theory point of view, which can in turn be used for testing irreducibility of a given matrix. An irreducible matrix is defined as follows: if A is irreducible (and hence indecomposable), then for all $i \neq j$, there is in D_A (digraph of A) a path from i to j. That is, in the digraph of the matrix, if the directions of the connections are followed, then there is a path from any node to any other node.

A reducible matrix is defined thus: a square $n \times n$ matrix $A = a_{ij}$ is reducible if the indices $1, 2, \cdots, n$ can be divided into two disjoint nonempty sets $i_1, i_2, \cdots, i_\mu$ and $j_1, j_2, \ldots, j_\nu$ (where μ and ν can take any positive value as long as $\mu + \nu = n$) such that $a_{i_\alpha j_\beta} = 0$.

With this setup, we can devise a programmable set of conditions for the color test in an irreducible matrix as follows:

For $ct1$, each (i, i) element that is negative is a black node. Let us denote these black node elements as $a_{bi,bi}$. (Note that the case of no negative diagonal elements does not lead us to the color test.)

For $ct2$, each (i, i) element that is zero is a white point. Let us denote these white node elements as $a_{wi,wi}$. Passing this condition of the color test implies there is at least one white node (i.e., there is at least one diagonal element that is zero). If there are no zero elements on the diagonal, it implies that this condition (and thus the color test) failed.

For $ct3$, form all products of the form $a_{wi,wj}a_{wj,wi}$. Passing this condition of the color test implies that at least one of these products is negative. If there is only one white node (in which case, there is no indicated product possible), then it implies that this condition (and thus the color test) failed. Similarly, if there is only one product possible (e.g., when there are only two white nodes), then that product being negative constitutes passing this condition of the color test.

For $ct4$, form all products of the form $a_{bj,wi}a_{wi,bj}$. Passing this condition of the color test implies that if the product $a_{bj,wi}a_{wi,bj}$ is negative for each fixed bj black node, then another product $a_{bj,wk}a_{wk,bj}$ is also negative for some $wk \neq wi$. If there is only one product possible, then passing this condition implies that this product is negative. If the products formed under this $ct4$ condition are *all* zero or *all* negative, it implies passing this condition.

Remark 1: Note that an irreducible matrix that fails the color test is a sign-stable matrix, but it is important to realize that there could be block triangular and block diagonal matrices that can be also sign-stable. Thus, the total number of sign-stable matrices of a given order consists of the combination of all irreducible matrices that are sign-stable and the set of reducible matrices (block triangular and block diagonal) that are sign-stable.

The test for sign stability of a given matrix can be illustrated best with the help of examples.

B. Examples Illustrating Sign Stability of a Matrix

In this section, we illustrate the notion of qualitative (sign) stability of a matrix by interpreting the preceding necessary and sufficient conditions in terms of matrix element notation and then decide whether that given matrix is qualitatively stable or not.

Example 1: Let us consider the following 3 by 3 sign matrix, given in [7]:

$$A = \begin{bmatrix} 0 & + & + \\ - & 0 & 0 \\ - & 0 & - \end{bmatrix}$$

First, let us test the necessary conditions $M1$, $M2$, $M3$, $M4$, and $M5$. Because a_{11} and a_{22} are zero and a_{33} is negative, conditions $M1$ and $M2$ are satisfied. Note that the product $a_{12}a_{21}$ is negative. Similarly, $a_{13}a_{31}$ is negative. Finally, $a_{23}a_{32}$ is zero. So condition $M3$ is satisfied. Next notice that $a_{12}a_{23}a_{31}$ is zero. Similarly, $a_{13}a_{32}a_{21}$, $a_{21}a_{13}a_{32}$,; $a_{23}a_{31}a_{12}$, $a_{31}a_{12}a_{23}$, and $a_{32}a_{21}a_{13}$ are all zero, and thus condition $M4$ is satisfied. It is easy to observe that det A is not zero, and thus condition $M5$ is satisfied as well.

Now we need to look at the color test. Noting that the preceding matrix is irreducible, we proceed with the application of color-test conditions. Note that we have only one self-regulating node: namely, a_{33} (which is negative). Thus, node (3,3) is black and nodes (1,1) and (2,2) are white. Thus, there are two white nodes. The observations up to this point pass conditions $ct1$ and $ct2$ of the color test. There is one predation community [namely, (1,2,3)] with two predation links [(1,2);(2,1)] and [(1,3);(3,1)]. So we form the product $a_{12}a_{21}$, which is already seen to be negative. Thus, condition $ct3$ of color test passes. Finally, for testing condition $ct4$, we form the products a_{31} a_{13}, and $a_{32}a_{23}$, out of which the former is negative, but the latter is zero. Thus, condition $ct4$ is not satisfied because the black node (3,3) is not connected to one of the white nodes (2,2) (because $a_{23} = a_{32} = 0$). That means that this matrix fails the color test. Thus, we can conclude that the *preceding matrix is qualitatively stable*.

Note that by the preceding logic, the sign matrix obtained by interchanging the first row and column, namely, the matrix

$$A = \begin{bmatrix} 0 & - & - \\ + & 0 & 0 \\ + & 0 & - \end{bmatrix}$$

is also qualitatively stable.

In fact, using the preceding logic, all the 3 by 3 matrices that are sign-stable may be determined a priori and tabulated. This is done by generating all the possible sign-pattern matrices and subjecting them through the necessary and sufficient conditions developed in this Note using matrix theory notation. This procedure yields 427 matrices (out of the 19,683 sign combination matrices possible) that are qualitatively stable. This number includes the reducible and irreducible matrices that are sign-stable. *It is important to realize that among all these sign-stable matrices, we can substitute any numerical values for the magnitudes of those entries and be guaranteed Hurwitz stability without the need for any computations. This is where the strength of the sign stability of ecology lies.*

III. Control Design via Ecological Sign Stability: Applications to Aerospace Flight Control

In this section, we consider the objective of synthesizing a controller for a linear state-space system such that the closed-loop system possesses the sign-stability property. As mentioned, this task is motivated by the observation that the matrix family that is sign-stable allows for robust stability of the system with arbitrarily large variations in the sign-preserving entries of the system. The control design methodology is demonstrated with an application in the aerospace flight control area.

Let us first consider the problem of satellite attitude stabilization [16] by a three-axis controller, encountered in satellite attitude control problems. It is well known that the linear range dynamics of an axisymmetric satellite spinning about one principal axis (say, the z axis) with a constant angular velocity Ω is given by

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{1}$$

where $x(t) \in R^3$ is the state vector consisting of the three state variables (namely, $x_1 = \omega_x$, the angular velocity about the principal axis x, $x_2 = \omega_y$, the angular velocity about the principal axis y, and $x_3 = \omega_z - \Omega$), and $u(t) \in R^3$ is the control vector consisting of the three control torques T_x, T_y, and T_z. Note that the entries of the preceding A matrix consist of elements as described next:

$$A = \begin{bmatrix} 0 & \kappa_1 & 0 \\ \kappa_2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

where $\kappa_1 = [(I_y - I_z)/I_x]\Omega$ and $\kappa_2 = [(I_z - I_x)/I_y]\Omega$, and the control distribution matrix B is given by

$$B = \begin{bmatrix} 1/I_x & 0 & 0 \\ 0 & 1/I_y & 0 \\ 0 & 0 & 1/I_z \end{bmatrix}$$

If we now employ a three-axis controller with the control law

$$u(t) = Gx(t) \tag{2}$$

where G is the control-gain matrix, then the closed-loop system matrix is obtained as

$$A_{cl} = A + BG = \begin{bmatrix} g_{11}/I_x & \kappa_1 + g_{12}/I_x & g_{13}/I_x \\ \kappa_2 + g_{21}/I_y & g_{22}/I_y & g_{23}/I_y \\ g_{31}/I_z & g_{32}/I_z & g_{33}/I_z \end{bmatrix}$$

where g_{ij} are elements of the control-gain matrix G. Then, as outlined in the previous section, it is possible to select the gain matrix elements such that the closed-loop system matrix possesses sign-stability property. Once this is done, it is clear that the resulting closed-loop system is robustly stable for any uncertainty in the angular velocity Ω. Let us illustrate this with the following application example. Let the moments of inertia for a hypothetical satellite be given by $I_z = 115$, $I_y = 104$, and $I_x = 96$ units, with a constant angular velocity $\Omega = 100$ units. Thus, $\kappa_1 = -11.46$ and $\kappa_2 = 18.27$. Then the closed-loop system matrix is given by

$$A_{cl} = A + BG = \begin{bmatrix} g_{11}/96 & (-11.46) + g_{12}/96 & g_{13}/96 \\ 18.27 + g_{21}/104 & g_{22}/104 & g_{23}/104 \\ g_{31}/115 & g_{32}/115 & g_{33}/115 \end{bmatrix}$$

Now we can select the gains of the preceding controller from sign-stability criteria. There can be many choices for carrying out the design. We can first select a 3 by 3 sign matrix that possesses the sign-stability property and then select the gains such that the preceding closed-loop system matrix possesses that particular sign pattern. For example, we can consider any one of the many sign-stable matrices such as

$$A_{\mathrm{cl}} = \begin{bmatrix} 0 & - & - \\ + & 0 & 0 \\ + & 0 & - \end{bmatrix}$$

or

$$A_{\mathrm{cl}} = \begin{bmatrix} 0 & + & + \\ - & - & 0 \\ - & 0 & - \end{bmatrix}$$

or

$$A_{\mathrm{cl}} = \begin{bmatrix} - & + & + \\ - & - & 0 \\ - & 0 & - \end{bmatrix}$$

Of course, it is understood that out of these many available sign-stable matrices (and their sign patterns), some are more desirable than others from various control design specifications and control objectives. Also it is possible that some sign patterns are not suitable in the sense of the existence of a controller, because for a given B matrix, there may not exist a suitable sign pattern of a sign-stable matrix that guarantees the existence of full state feedback control gain G. Systematic conditions under which the existence of a controller gain G is guaranteed for given open-loop system matrices A and B are beyond the scope of this Note, as the main aim of this Note is to highlight the possible use of sign stability in designing controllers. The development of systematic design guidelines to address many control design specifications such as performance, control effort, physical limitations of control effectors, etc., is part of an ongoing research effort, and preliminary results on these issues are being presented in various conferences [15,17].

For illustrating the usefulness of the proposed sign-stable matrix theory approach, suppose that we now select the first matrix among these as our desired sign-pattern matrix. Then it is clear that the gain elements need to be selected such that $g_{11} = g_{22} = g_{23} = g_{32} = 0$. It is interesting to observe that even the gains g_{12} and g_{21} can be selected to be zero, which in turn helps in reducing the control-gain norm. If one uses control-gain norm as a measure of control effort, this implies that ecological sign-stability-based controllers may produce controllers of reduced control-gain norms. Note that g_{13} can be any gain that is negative and g_{31} can be any gain that is positive. Finally, g_{33} can be any gain that is negative. Note that we do have control over the selection of magnitudes of these gains. Thus, the final robust control-gain matrix is given by

$$G = \begin{bmatrix} 0 & 0 & -* \\ 0 & 0 & 0 \\ +* & 0 & -* \end{bmatrix}$$

where the stars indicate that the magnitudes of those entries are arbitrary. Once we select these gains, it is clear that the proposed controller is robust for arbitrarily large sign-preserving variations in the constant angular velocity Ω, as well as arbitrarily large sign-preserving variations in the parameters κ_1 and κ_2.

With the preceding selection of gains, a very interesting observation emerges. Note that the entire second row of the gain matrix is zero, indicating that the control torque about the y axis is completely unnecessary. This implies that we can require only two actuators: one generating a control torque about the x axis and the other about the z axis. Thus, the control design based on qualitative (sign) stability produces not only a robust controller, but also a controller with a reduced number of inputs. Also, the fact that the second control input is unnecessary is brought out in a very transparent way. Note that if we design a linear quadratic regulator (LQR) controller for the preceding system with a typical choice of state and control weighting matrices, the resulting controller is still a three-axis controller, whereas the controller based on sign stability produced a two-axis controller that robustly stabilizes the closed-loop system under arbitrarily large variations in the spin angular velocity and sign-preserving variations in the parameters κ_1 and κ_2. It may be argued that in the LQR design, if we select the weighting matrices in a more appropriate way (e.g., a very large penalty on one of the control inputs), one may also obtain a controller with a reduced set of control inputs. However, that argument is misleading because in the sign-stability design, the role of individual gain matrix elements is transparent and is known a priori, whereas the LQR design does not recognize the unnecessary nature of the second control input a priori. The argument that one can reduce the number of control inputs needed to control the system by selecting very large weightings on the control inputs does not always work, especially if one is trying to control an unstable open-loop plant. Thus, in this example, sign-stability controller does have an advantage over the LQR controller.

It is indeed well known in the aerospace controls community that the preceding three-axis-controlled satellite attitude is very robust with respect to variations in the spin angular velocity and even for variations in the moments of inertia. Thus, the robust stabilization scheme based on the sign-stability concept corroborates that observation. The three-axis satellite attitude spin-stabilization problem presented here helps to simply illustrate the utility of the sign-stability concept in the robust stabilization problems. Indeed, using the ideas presented in this Note, it is possible to propose a robust stabilization scheme based on sign stability for other flight control problems as well. An aircraft flight control application is discussed in [17].

IV. Conclusions

This Note presents a new control design method for linear uncertain time-invariant state-space systems that is robust with respect to a class of plant parameter variations using the sign-stability concept from ecology. It is then shown that the resulting closed-loop system is robust with respect to all sign-preserving plant parameter variations. It is also shown that the control design via the ecological sign-stability approach produces controllers of reduced control-gain norm, as we have control over the selection of magnitudes of the control-gain elements. The strengths of the proposed methodology lie in computational simplicity and transparency of the control design algorithm, and the current weakness is that it is more attractive for only low-order dynamic systems.

References

[1] Lotka, A. J., *Elements of Physical Biology*, Williams & Wilkins, Baltimore, MD, 1925

[2] Volterra, V., "Principes de Biologic Mathématique," *Acta Biotheoretica*, Vol. 3, 1937, pp. 1–36.

[3] Connell, J., and Sousa, W., "On the Evidence Needed to Judge Ecological Stability or Persistence," *The American Naturalist*, Vol. 121, No. 6, 1983, pp. 789–824.
doi:10.1086/284105

[4] May, R., "Will a Large Complex System be Stable?," *Nature*, Vol. 238, 1972, pp. 413–414.
doi:10.1038/238413a0

[5] Dambacher, J., Luh, H., Li, H., and Rossignol, P., "Qualitative Stability and Ambiguity in Model Ecosystems," *The American Naturalist*, Vol. 161, No. 6, June 2003, pp. 876–888.
doi:10.1086/367590

[6] Lunghini, G., "Qualitative Analysis, Determinacy and Stability," *Quality and Quantity*, Vol. 4, No. 2, 1970, pp. 299–324.

[7] Edelstein-Keshet, L., *Mathematical Models in Biology*, McGraw–Hill, New York, 1988, pp. 236–241.

[8] Quirk, J., and Ruppert, R., "Qualitative Economics and the Stability of Equilibrium," *The Review of Economic Studies*, Vol. 32, 1965, pp. 311–326.
doi:10.2307/2295838

[9] May, R., *Stability and Complexity in Model Ecosystems*, Princeton Univ. Press, Princeton, NJ, 1973, pp. 70–74.

[10] Jeffries, C., "Qualitative Stability and Digraphs in Model Ecosystems," *Ecology*, Vol. 55, 1974, pp. 1415–1419.
doi:10.2307/1935470

[11] Jeffries, C., Klee, V., and Driessche, P., "Qualitative Stability of Linear Systems," *Linear Algebra and Its Applications*, Vol. 87, 1987, pp. 1–48.
doi:10.1016/0024-3795(87)90156-X

[12] Jeffries, C., Klee, V., and van den Driessche, P., "When is a Matrix Sign Stable?," *Canadian Journal of Mathematics*, Vol. 2, No. 29, 1977, pp. 315–326.

[13] Klee, V., and van den Driessche, P., "Linear Algorithms for Testing the Sign Stability of a Matrix and for Finding Z-Maximum Matchings in Acyclic Graphs," *Numerische Mathematik*, Vol. 28, 1977, pp. 273–285.

[14] Kaszkurewicz, E., and Bhaya, A., *Matrix Diagonal Stability in Systems and Computation*, Birkhauser, Boston, 2000, pp. 33–80.

[15] Yedavalli, R. K., "Qualitative Stability Concept from Ecology and Its Use in the Robust Control of Engineering Systems," *Proceedings of the American Control Conference*, Inst. of Electrical and Electronics Engineers, Piscataway, NJ, June 2006, pp. 5097–5102.

[16] Yedavalli, R. K., "Robust Satellite Attitude Control Using Qualitative Stability Concept from Ecology," Proceedings of the AIAA Guidance, Navigation, and Control Conference, San Francisco, AIAA Paper 2005-6262, Aug. 2005.

[17] Yedavalli, R. K., and Devarakonda, N., "Ecological Sign Stability and Its Use in Robust Control Design for Aerospace Applications," *Proceedings of the Conference on Control Applications* (to be published).